THE EÖTVÖS EXPERIMENT

With 2 figures

by

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Received 28. 9. 1963.

SUMMARY

Among the significant scientific works of Roland Eötvös his investigations concerning the proportionality between the attraction of masses and the inertia are of a special importance from the view point of modern scientific development. By means of these investigations he has proved, that even if there were a difference in the attraction of materials of various quality, it ought to be smaller than 1/200 000 000. Subsequent investigators have exaggerated this accuracy. The accuracy has been raised by one order of magnitude in the 1930 by the author of this paper. In the last years it was found by R. H. Dicke that the attraction of gravity is independent from the quality of materials to an accuracy as high as $1/10^{11}$. In connection with his experiments Eötvös has also investigated the absorption of massattraction with his very sensible compensator and at his experiments of high accuracy he has not found any absorption.

The various significant scientific works of Roland Eötvös are usually denoted by brief characteristic denominations. Thus we speak about the Eötvös law of surface tension, the Eötvös effect expresses the gravitational changes of moving bodies and mainly recently his significant investigations concerning the proportionality between gravitational attraction and inertia are referred to as Eötvös experiment. It is well known for the professional that Eötvös and his collaborators Dezső Pekár and Jenő Fekete won the Benecke Prize of the University of Göttingen in 1909 with their paper describing these investigations. The prize-winning paper was not published by Eötvös himself, it was published by his collaborators as a posthumous work several years after his death in the 1922 volume of Annalen der Physik. This work contains the prize-winning paper in a slightly abridged form. The original paper was published to somewhat fuller extent in the collection "Eötvös's Gesammelte Arbeiten", published in 1953 edited by Pál Selényi.

The investigations, that served as a basis for the paper, were carried out by Eötvös and his collaborators around 1900, his investigations concerning this object go back, however, to earlier times, they are virtually equal in age with his gravity researches. A document of his earlier investigations on this object is his report in the Academy of Sciences of Hungary on 20. th of January 1889, the material of wich appeared in 1890 in Hungarian and German.

The problem is a very old one, since it contains the question whether the acceleration depends on composition of materials in the gravitational field. This question was answered by the classical falling experiments of Galilei and later by the observations of the swinging time on pendulums, loaded by various materials, conducted by Newton. These experiments lead to the conclusion, that the gravitational attraction is independent from the composition of materials.

The experiments conducted by excellent investigators during centuries showed a gradual development, since the accuracy of the measurements constantly improved. The accuracy of Newton's experiments achieved 1/1000, in the first half of the 19-th century the experiments of Bessel with pendulums of various materials achieved the accuracy of 1/50 000.

In the 80-es of the last century Eötvös used the torsion balance constructed by him for his experiments concerning the proportionality between gravitational attraction and inertia and the same method was used by him with an improved process and greater accuracy for his investigations carried out at the beginning of our century. According to his report on the Academy in 1889 the accuracy achieved at that time was 1/20 000 000, while the accuracy of the experiments described in the prize-winning paper of Göttingen reached 1/200 000 000, i.e. it was 10 times higher. In his former experiments Eötvös tested brass, glass, antimonite and cork. In the experiments for the prize work of Göttingen he tested the following materials: magnalium, snakeweed, copper, water, crystalline cupric sulfate, solution of cupric sulfate, asbestos, tallow, silver sulfate and iron sulfate. The above mentioned materials were compared with platinum. The experiments on the proportionality between gravitational attraction and inertia as conducted by Eötvös were founded on an original principle, completely differring from the earlier ones, therefore they deserve the name "Eötvös experiment". The principal basis of Eötvös investigations is, that since the centrifugal force from the rotation of the Earth – as an inertial force – is independent from the material composition, if the gravitational attraction caused by the Earth were different for various materials, then the resultant, i.e. the gravity would change according to the quality of materials and the direction of gravity would also be different for various materials. By other words it means that for every kind of material a different niveau surface would form. The torsion balance of Eötvös is excellently suitable for measuring very small deviations in the direction of gravitational force.

In the followings I shall briefly decribe Eötvös's method. On fig. 1. ε is the angle between the force of attraction and the direction of resultant gravitational force. η is the assumed deviation of direction caused by differences of material composition. The value of angle ε depends on geographical latitude and its maximum is about 6' on the latitude of 45°. From the triangle on the figure PBB_1 $\eta = \dfrac{G_1 - G}{g} \sin \varepsilon$, where, G and G_1 represent the varoius accelerations according to material composition.

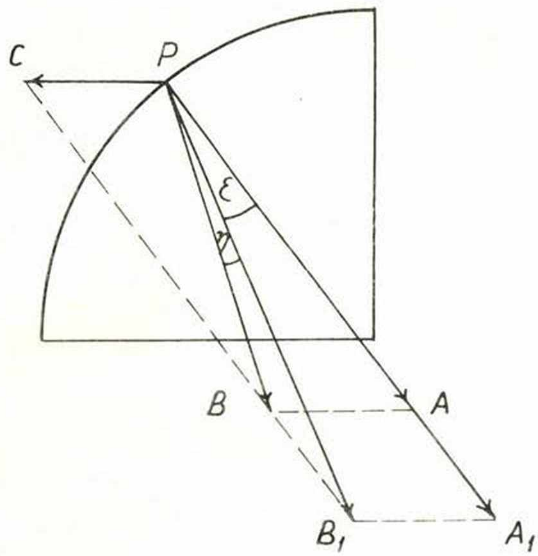

Fig. 1.

Let us assume, that $G_1 - G = \varkappa G$, hence
$\varkappa$ is a factor depending on material composition.

Let us assume, that the ends of the beam of the torsion balance are loaded with weigths of various compositions, and the corresponding factors of material composition is $\varkappa_a$ and $\varkappa_b$. The horizontal torque arising from the difference of material composition in the position of the beam characterized by the angle α is:

$$(M_b l_b \varkappa_b - M_a l_a \varkappa_a) G \sin \varepsilon \sin \alpha$$

In this expression M_a and M_b are the weights loading the balance, l_a and l_b are the corresponding arms of rotation.

The condition of equilibrium as known from the theory of the torsion balance, adapted for the horizontal variometer and considering the assumed difference due to material composition is

$$\vartheta = \frac{1}{2}\frac{K}{\tau} U_{\Delta} \sin 2\alpha + \frac{K}{\tau} U_{xy} \cos 2\alpha - \frac{M_a l_a h}{\tau} U_{xz} \sin\alpha +$$

$$+ \frac{M_a l_a h}{\tau} U_{yz} \cos \alpha + \frac{M_a l_a}{\tau} (\varkappa_b - \varkappa_a) G \sin \varepsilon \sin \alpha$$

In this formula K is the moment of intertia of the suspended system, τ is the torsional moment of the wire, h is the distance from the centre of the lower suspended weight to the horizontal plain of the beam. U_{xz}, U_{yz}, U_{Δ} and U_{xy} are the second derivatives, characterising the gravitational field.

The angle ϑ is the deviation from the untorqued position of equilibrium.

In these investigations Eötvös used several methods. In his first method he assumed, that the field of gravitational force and the torque of the wire are constant. In the second one he assumed the constancy of the gravitational field, but allowed a slow change for the torque of the wire. In the third most accurate method neither the constancy of the gravitational field nor that of the torsion wire had to be assumed. In the third method Eötvös used double torsion balance with antiparallel beams. The experiment was arranged so, that the upper weights of the beam were made of the same material with $\varkappa_b$ characteristic coefficient, the lower suspended weights were made of the materials of various composition, selected in comparison with coefficients $\varkappa_a$ and $\varkappa'_a$. The positions of the double balance were observed in azimuthal angles shifted for 90°. Let us mark the difference of readings, obtained in positions of $E-W$ by v, the difference of readings taken in positions $N-S$ by m and be $\Delta\alpha$ the very little angle closed in initial position by the axis of the beam and the astronomical North.

Two series of observations are needed: at first to the end of the first beam is attached a weight with coefficient $\varkappa_a$, next to the second beam another weight with coefficient $\varkappa'_a$. Then the corresponding values for the first beam are v_1, m_1, $\Delta\alpha_{1I}$, for the second beam v'_2, m'_2, $\Delta\alpha_{2I}$. At the second series of observations the suspended weights are mutually changed, then the values for the first beam are v'_1, m'_1, $\Delta\alpha_{1II}$, and the values for the second beam are v_2, m_2, $\Delta\alpha_{2II}$.

The ratio of the quantities v and m is not subjected neither to the casual changes of sensibility, nor to the slow change of the gravitational field. From the above mentioned position of equilibrium one can compute the difference of coefficients, characterising the material composition:

$$\varkappa_a-\varkappa'_a=\frac{m\tau}{8LM_a l_a G\sin\varepsilon}\left\{\left(\frac{v_1}{m_1}-\frac{v'_2}{m'_2}\right)+\left(\frac{v_2}{m_2}-\frac{v'_1}{m'_1}\right)\right\}+$$

$$+\frac{m\tau}{8LM_a l_a G\sin\varepsilon}\left(1+\frac{v^2}{m^2}\right)\{(\Delta\alpha_{1I}-\Delta\alpha_{1II})-(\Delta\alpha_{2I}-\Delta\alpha_{2II})\}$$

In this formula L is the distance of the scale.

In connection with these experiments Eötvös examined very carefully the achieved accuracy. It can be seen from the above formula, that if there were any difference in the gravitational attraction of various materials, it ought to be appear in the value of the ratio v/m. Since the values v and m were directly observed, according to the rules of error propagation the mean error of the mean values of the quantity v/m from the mean errors of the observed mean values is:

$$\Delta\left(\frac{v}{m}\right)=\sqrt{\left(\frac{v}{m^2}\Delta m\right)^2+\left(\frac{1}{m}\Delta v\right)^2}$$

The mean errors Δv and Δm were calculated by Eötvös in the usual way with the formula $\sqrt{\frac{\Sigma\delta^2}{n(n-1)}}$.

The individual series of observation consisted of about 100 readings, and since within the individual series the values v and m had a minute of any change, the δ deviations as calculated from the mean values were very little and in consequence of this the mean errors Δv and Δm of the mean values proved to be very small.

This can be expected, of course, if any disturbing circumstance is eliminated, which may change the position of the beam of the torsion balance. Eötvös paid particular attention to the elimination of disturbances like these. He used carefully prepared and checked wires in the torsion balance. He cancelled the occasional electrostatic and magnetic influences and created a temperature protection for the instrument that no disturbances due to changes in temperature could take place. The mentioned disturbing effects may appear in the values for v and m, and if they were not eliminated in a corresponding manner, they might cover the effect, that may be caused by the differences of material compositions in the gravitational attraction. The order of magnitude of the accuracy achieved by Eötvös are characterised by the following data. In the series of about 100 observations the mean errors of the mean values of the quantities v and m are generally of $\pm 0{,}01$ scale division. According to the rules of error propagation the mean errors of the ratios v/m are of the order of about $\pm 0{,}002$; the error of the sum of the differences of ratios v/m is about $\pm 0{,}004$. If this is multiplied by the outer factor of appr. $0{,}4 \cdot 10^{-6}$, the mean error has the value of about $\pm 0{,}0016 \cdot 10^{-6}$. Let us take into consideration, that in the several series of Eötvös experiments the investigated material was placed is some container, metallic or glass tube and therefore the error must be related only to the part of mass made of the investigated material. In consequence of this the mean error increases to about $\pm 0{,}003 \cdot 10^{-6}$. The value of the differences $\varkappa_a - \varkappa_a'$ was determined from the observations to be $0{,}001 \cdot 10^{-6}$—$0{,}006 \cdot 10^{-6}$, thus they are partly bigger and partly smaller, than the computed average errors, their order of magnitude, however, corresponded to the order of magnitude of the mean errors. From these results Eötvös drew the conclusion, if there were any deviation between the gravitational attractions of various materials, it ought to be smaller than the value of $0{,}005 \cdot 10^{-6}$.

In such a way the accuracy, achieved by Eötvös was 1/200 000 000.

Apart from the investigation of the above mentioned materials Eötvös conducted special experiments with radioactive materials. The age, in which Eötvös carried out his very accurate experiments on the proportionality between gravitational attraction and inertia, witnessed the discovery of radioactive phenomena. At that time R. Geiger maintained the idea, that the radioactive radiation absorbs the energy of attraction. Eötvös placed a radium-compound in a little glass tube in the proximity of the platinum weight on the beam of the torsion balance and experienced a little effect of repulsion or attraction depending on the position of the glass tube. The experiment was repeated in such a way, that the glass tube containing the radium-compound had been replaced by another glass tube, in which a thin platinum wire was welded and connected in an electric current. When the quantity of heat, produced in the platinum wire by electric current, was equal to that, performed by the radiation of the radium-compound, the effect of repulsion and attraction was observed to be the same, thus the phenomenon was completely explained by the temperature effect. There was no trace of the absorption of gravitational attraction.

Following the investigations of Eötvös, in the years 1930–1935 I have also dealt with the question of proportionality between gravitational attraction and inertia. My endeavour was to increase further the accuracy and to extend the investigations to various materials, not included in experiments of Eötvös. For the purpose of the experiments I have used the double torsion balance No. III of the Geophysical Institut, which at that time proved to be the most reliable laboratory instrument. It was especially remarkable for being almost unsensible to temperature influences. The torsion wires were long ago prepared platinum – iridium wires, the temperature coefficient of which was practically zero and had no elastical drift at all. A special care was taken of the constant temperature of the surroundings. I have compensated the geomagnetic field similarly to Eötvös and apart from this I have used the very observations for determining the occasional magnetic effect. This was achieved by carrying out the observations besides the chief directions of E–W and N–S also in intermediary positions making an angle of 45° with these directions. From the latter the occasional magnetic effect could be computed. This method was used by me at the comparison of brass and diamagnetic bismuth. Otherwise the observations in the intermediary positions of 45° represent a series in themselves, from which the differences $\varkappa_a - \varkappa_a'$ can be determined independently from the result of the observations in the main positions, thus they serve as a control. This investigation was carried out at the comparison of copper and an alloy of manganous copper and from the two series of observations I have recieved the following independent results: in the main positions:

$$\varkappa_a - \varkappa_a' = +0{,}08 \cdot 10^{-9} \pm 0{,}20 \cdot 10^{-9}$$

in the intermediary positions:

$$\varkappa_a - \varkappa_a' = +0{,}12 \cdot 10^{-9} \pm 0{,}22 \cdot 10^{-9}$$

The differences of the coefficients characterising the material composition were within the limits of error in both series and the error of the mean values was about $\pm 0{,}2 \cdot 10^{-9}$, i. e. 1/5 000 000 000, that exceeds 25 times the accuracy of the experiments conducted by Eötvös.

At the subsequent experiments the order of mean errors was about $\pm 0{,}5 \cdot 10^{-9}$, i. e. 1/2 000 000 000, hence the accuracy exceeds in general the accuracy of the experiments of Eötvös 10 times.

Besides of the already mentioned materials the following materials were compared:

platinum – brass
batavian glass beads – brass
batavian scrap glass beads – brass
paraffin – brass
aluminium fluoride – copper.

The differences $\varkappa_a - \varkappa_a'$ computed from the series of observations had partly positive, partly negative signs, the mean errors were of the same order, their mean value being $\pm 0{,}52 \cdot 10^{-9}$.

Eötvös used another method for examining the proportionality between gravitational attraction and inertia, too. This method is founded on the comparison of the tidal force raised by the Sun and of the centrifugal force appearing on the orbit of the Earth. The torsion balance is very suitable for investigating this phenomena. Let us assume, that there are masses of various quality attached to the ends of the beam, orientated along the meridian, and that the Sun exerts a greater attraction to the mass on the northern end of the beam, than to the one on its southern end. In this case the mass on the northern end would move towards the Sun, when it rises and cause a corresponding turn of the beam; at Sunset the mass at the northern end would move towards west and the beam would turn in a direction opposite to the previous one. Thus owing to the different specific gravitational attraction the beam of the torsion balance would show up an oscillation with a 24-hour period, that could be observed with a balance of appropriate sensibility.

In comparison with the above discussed method the latter one has the advantage that the torsion balance remains in the same azimuthal position during the whole series of observations and the object of the observation must be the occasional change in the state of equilibrium. But the sensibility of this method is only a third part of the sensibility of the previous one.

Eötvös and his collaborators compared the coefficient of attraction of magnalium and platinum by the help of this method and the difference $\varkappa_a - \varkappa_a'$ was determined to be of the same order of magnitude, as in the experiments conducted with the other method. For the purpose of such experiments the highly sensitive instrument of Eötvös, the gravitational compensator is particulary suitable.

In the last years the same principle was used essentially for the investigation of the proportionality between gravitational attraction and inertia by the American investigator R. H. Dicke, who however had designed an instrument with modern technics for his experiments.

Dicke suspended a frame in the form of a triangle on three wires, and the corners of the triangle were loaded by weights of the same mass. Two of the three weights were made of copper, and the third was lead chloride sealed in a cylindrical flask. From one silvered side of the triangle the reflected light is brought through an adequate optical device to a wire oscillating at 3,000 cycles per second, from there it strikes a photocell having a certain current intensity at constant illumination, hence a constant current intensity corresponds to the unchanged position of the triangle. If the triangle turns but slightly, the signal from the photocell changes and gives rise to a direct-current voltage which by the help of a servo-mechanism exerts a restoring force on one of the copper weights, thus it makes the whole system to return into its initial position. The restoring force is registered and this measures the rotation angle of the suspended system. Dicke placed the whole system in a metal can and the air was evacuated from it to a pressure of 10^{-6} millimeter of mercury. By such a way he cancelled the disturbances from air convection. The whole apparatus was mounted in a chamber at a depth of 4 m. According to an information from him after having overcome many technical difficulties he succeded in improving on the accuracy of the experiments Eötvös by a factor of 50 and on the accuracy of my experiments conducted in the 1930-es by 5. According to latest informations the accuracy in Dicke's experiments raised further to the order of 10^{-11}.

The proposal of Pál Selényi deserves attention, according to which it would be worth-while to extend the investigations on living material.

In the following I should like point briefly at the significance of the Eötvös experiment from the viewpoint of the general theory of relativity, on the one hand, and of the modern atomic physics, on the other.

It is known, that the general theory of relativity of Einstein is based on the principle of equivalence, according to which the attractive and inertial masses are equal. It is probable, that Einstein did not know the results of Eötvös experiments as he formulated the theory and he became acquainted with them only afterwards. In any case, undependently from the result of the experiments Einstein was convinced of the correctness of the principle of equivalence. He endeavoured to determine the laws of motion for accelerating systems. Therefore he had to identify the inertial forces with the gravitational ones, that means, that in an accelerating system an observer closed from the outer world can not distinguish whether the motion of a certain mass is called forth by the gravitational field or by the inertia.

Recently the Eötvös experiment has obtained significance also in atomic physics. Namely among the many elementary particles known at present there are ones differring from one another only in the sign of their electric charge. Particles like this are the electron and positron, proton and anti-proton etc. There are so called anti-particles that appear mostly in the big accelerators and have a very short period of life. The English physicist Bondi created a hypothesis according to which the gravitational effect from anti-particles would be repulsing. According to this hypothesis the proportionality between gravitational attraction and inertia would not be valid between particles and anti-particles. Since the anti-particles appear most rarely in our world it seems impossible to examine experimentally the principle of equivalence. According to the opinion of the Californian investigator L. J. Schiff in the atoms of our world there can be also found anti-particles, since the electric fields in the inner part of the atom create virtual pairs of electron and positron, and if the positrons had a gravity with sign contrary to that of electrons, it ought to appear in the experiments of high accuracy concerning the proportionality of gravitational attraction and inertia. Nevertheless at the high accuracy achieved in these experiments there are no traces of a phenomenon like this, consequently there exists no antigravitation. In any case this problem contributed to bringing the experiments on the proportionality of gravitational attraction and inertia into prominence and to-day's investigators strive to improve further on the accuracy.

The investigations of Eötvös concerning the absorption of gravitational attraction are of great interest, too, the results of which were also described in the prize-winning paper of Göttingen. Eötvös used for these investigations the gravitational compensator, constructed by him. This was a very sensitive instrument and its sensitivity can be virtually raised beyond all limits by appropriately regulating the lead masses in the form of a quadrant, mounted in the proximity of the ends of the beam. If the central line of the pairs of lead quadrants are adjusted in 45° and 225° to the vertical, then one of the quadrants fälls under the horizontal plane, the other above it; if the centre lines are adjusted in 135° and 315°, those quadrants raise above the horizontal plane, that were previously below it and conversely. In each case the attraction from one half of the earth exerts its influence through the lead mass of the quadrant, and the attraction from the other half of the earth is not shaded by the lead mass. If the gravitational attraction had an absorption, then the gravitational attraction ought to be greater on that side, where is no shading mass. (Fig. 2.)

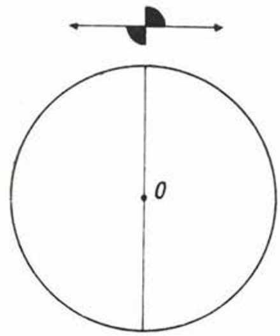

Fig. 2.

If the readings on the torsion balance in the subsequent positions of the quadrants are marked by n_1, n_2, n_3, n_4 then according to Eötvös computations the coefficient of absorption can be expressed as follows:

$$\mu = \frac{n_2 + n_4 - n_1 - n_3}{47890 \cdot 10^6} + \frac{608\zeta}{47890 \cdot 10^6}$$

where ζ means the vertical coordinate of the gravitational centre of the mass at the end of the beam in a coordinate system, the origin of which takes place on the horizontal line passing through the centre of the opposite quadrants. If the beam is adjusted centrically in relation to the quadrants, which must be achieved upon the orientation, then the term containing ζ in the above expression can be neglected.

According to the results of the performed experiments the numerator of the first term is of the order of unit, thus it can be stated if the gravitational attraction had an absorption, then the absorption of the lead quadrants of the compensator must be smaller than one in fifty milliards. This shading corresponds to a lead layer with a thickness of about 5 cm. Recalculated this for a lead layer of 1 m thickness the absorption would be smaller than one in two thousend five hundred millions. Calculated on this base the absorption of the whole earth along its diameter would be at the most one part in eight hundred.

If the absorption is applied to the tidal phenomena, then for the effect from the Sun the effect at the comparison of the zenith positions of 0° and 180° can be expressed as follows:

$$-Z = 2f \frac{M}{D^2} \cdot \frac{a}{D} (1 + 11800\mu)$$

In this expression a is the radius of the Earth, D — distance to the Sun; M — mass of the Sun. If, using the result of the experiments with the compensator, 1/1600 is substituted for μ, the tidal force created by the Sun is:

$$-Z = 2f\frac{M}{D^2}\cdot\frac{a}{D}\,(1+7{,}4)$$

i. e. the absorption would increase the tidal force 8 times, which fully contradicts the experiences. According to this the above mentioned absorption is impossible.

The scientific activity of Roland Eötvös, our great scientist, embraces a wide scope of problems in the field of gravity, the results achieved by him have a basic significance. The problems, he dealt with so extensively and successfully are actual even at present after a half of a century, they inspire thoughts in the investigators of today and stimulate them to further investigations.